Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (OLD) EXAMINATION – SUMMER 2022

Subject Code:140001 **Date:23-06-2022**

Subject Name:Mathematics-IV

Time:10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Find and graph all sixth roots of unity. **07**

(b) Expand $f(z) = \frac{1}{(z+2)(z+4)}$ vaild for the region (i) $|z| < 2$ (ii) $2 < |z| < 4$ (iii) $|z| > 4$. **07**

Q.2 **(a)** Using the residue theorem, evaluate $\int_C \frac{e^z+z}{z^3-z} dz$, where $C: |z| = \frac{\pi}{2}$ **07**

(b) If $f(z) = u + iv$ is analytic in domain D then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|Ref(z)|^2 = 2|f'(z)|^2$. **07**

OR

(b) Evaluate $\int_C \bar{z}\, dz$, where C is along the sides of the triangle having vertices $z = 0, 1, i$. **07**

Q.3 **(a)** Explain bisection method for solution of equation using this method find the approximate solution of $x^3 - x + 1 = 0$ correct up to three decimal points. **07**

(b) Apply fourth order Range-kutta method to find $y(0.2)$ given $\frac{dy}{dx} = x + y, y(0) = 1$. (Taking $h = 0.1$) **07**

OR

Q.3 **(a)** State Trapezoidal rule with n =10 and using it, evaluate $\int_0^1 2e^x\, dx$ **07**

(b) Expand $\frac{1}{z(z^2-3z+2)}$ about z = 0, for the regions **07**

(i) $1 < |z| < 2$ (ii) $|z| > 2$.

Q.4 **(a)** Solve the following system of equations using Gauss seidel method: **07**

$5x + y - z = 10$; $2x + 4y + z = 14$; $x + y + 8z = 20$

(b) Find the bilinear transformations which maps the points 1, -1, ∞ onto the points $1 + i, 1 - i, 1$ respectively. Also, find its fixed points. **07**

OR

Q.4 **(a)** Determine the polynomial by Newton's forward difference formula from the following table: **07**

X:	0	1	2	3	4
Y:	-10	-8	-8	-4	40

Also find y when x = 1.5

(b) Evaluate $\int_{0.2}^{1.4}(sinx - logx + e^x)dx$ with h=0.2 by simpson's 1/3rd and 3/8th rule. **07**

Q.5 **(a)** Find a real root of $xe^x = 2$, correct up three decimal places, by using Newton-raphson method **07**

(b) Evaluate following integrals using residue: **07**

$\oint_c \frac{2z^2+3}{z(z+1)(z+2)} dz$, where C: |z|=1.6

OR

Q.5 **(a)** Find the analytic function $f(z) = u + iv$, if $u - v = e^x(cosy - siny)$ **07**

(b) Apply Gauss Jacobi method to solve system of linear equation as under: **07**

$$20x + 2y + z = 30$$
$$x - 40y - 3z = -75$$
$$2x - y + 10z = 30$$

* * *